

SUPPLY CHAIN INVENTORY MODEL WITH VARIABLE DEMAND

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Abstract: The present paper presents a supply chain inventory model. Inventory is held in each of the stages to support the requirement of the customer at the end of the chain. The demand is taken as variable illustrated by three successive time period classified time dependent ramp type function. Numerical example is illustrated to test the model.

Keywords: Demand, Supply Chain, Inventory.

1. INTRODUCTION

One of the most important topics in the study of the management of contemporary manufacturing and distribution is supply chain management (SCM). Inventory management is an essential process for all parties engaged in supply chain activities, from the procurement of raw materials through to the delivery of finished goods. The effective execution of this process has a major influence on both the financial and operational performance of an organization.

The demand pattern for fashionable products which initially increases exponentially with time for a period of time after that it becomes steady rather than increasing exponentially. But for fashionable products as well as for the seasonal products the steady demand after its exponential increment with time never be continued indefinitely. Rather it would be followed by exponential decrement with respect to time after a period of time and becomes asymptotic in nature. Thus the demand would be illustrated by three successive time period classified time dependent ramp type function, in which in the first phase the demand increase with time and after that it becomes steady and towards the end in the final phase it decreases and becomes asymptotic.

Goyal and Nebebe (2000) considered a problem of determining economic production from a vendor to a buyer. **Wee (2003)** developed an integrated inventory model with constant rate of deterioration and multiple deliveries. **Lee and Wu (2006)** developed a study on inventory replenishment policies in a two-echelon supply chain system. **Ahmed et. al (2007)** have recently coordinated a two level supply chain in which they considered production interruptions for restoring of the quality of the production process. **Singh (2008)** assumed optimal ordering policy for decaying items under inflation.

Researches on real market oriented time dependent demand is very restrictive. **Hill (1995)** resolved the indiscipline of time dependent demand pattern by considering the demand as the combination of two different types of disciplined demand in two successive time periods over the entire time horizon and termed it as ramp – type time dependent demand pattern. **Wu (2001)** considered an EOQ model with ramp – type demand, weibull distribution deterioration and partial backlogging. The characteristic of ramp – type demand can be found in **Mandal and Pal (1998)** has been taken order level inventory system with ramp–type demand rate for deterioration items. **Wu et al (1999)** developed an EOQ model with ramp type demand rate for items with Weibull deterioration. **Wu and Ouyang (2000)** considered a replenishment policy for deteriorating items with ramp type demand rate, **Manna and Chaudhari (2006)** assumed an EOQ model with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages and **Deng et al. (2007)** considered a note on the inventory models for deteriorating items with ramp type demand rate.

In this model we consider only order cycle starts and ends with in the time interval $[\mu_i, \gamma_i]$. We believe that our study will provide a solid foundation for the further study of this sort of important inventory models with variable demand.

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Further, we analytically identify the best circumstance among these special cases based on the minimum total relevant cost per unit time. Finally, numerical examples are presented to demonstrate the developed model and the solution procedure.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations are used in developing the model

- (i) Shortages in the inventory are allowed and partially backlogged.
- (ii) The supply is instantaneous and the lead time is zero.
- (iii) The deterministic demand rate R(t) is ramp-type time dependent i.e.,

$$D(t) = Ae^{b[t - (t - \mu_i)H(t - \mu_i)] - [(t - \gamma_i)H(t - \gamma_i)]}, i = 1, 2$$

Where, $A > 0$ is the initial demand rate and $b > 0$ is the rate with which the demand rate increase. $H(t - \mu_i)$ and $H(t - \gamma_i)$ are well known Heviside functions respectively .

- (iv) A deteriorated unit is not repaired or replaced during a given cycle.
- (v) Single vendor and single buyer model is considered.

3. MODEL FORMULATION

The minimum unit running cost of the system is determined in this case if the order cycle starts in the period $[0, \mu_i]$ but ends in the time period $[\mu_i, \gamma_i]$. The demand is an increasing function of time up to time and after that it becomes steady. However it may be noted that there may arise two situations: (a) and (b). The behaviour of the instantaneous inventory level for both the cases it is governed by the differential equations:

$$I_{v1}(t_1) + \theta I_{v1}(t_1) = (K - 1) A e^{bt_1}, 0 \leq t_1 \leq \mu_1 \tag{1}$$

$$I_{v1}(t_1) + \theta I_{v1}(t_1) = (K - 1) A e^{b\mu_1}, \mu_1 \leq t_1 \leq \gamma_1 \tag{2}$$

$$I_{v2}(t_2) + \theta I_{v2}(t_2) = -A e^{bt_2}, 0 \leq t_2 \leq \mu_2 \tag{3}$$

$$I_{v2}(t_2) + \theta I_{v2}(t_2) = -A e^{b\mu_2}, \mu_2 \leq t_2 \leq \gamma_2 \tag{4}$$

$$I_b(t) + \theta I_b(t) = -A e^{bt}, 0 \leq t \leq \mu_2 \tag{5}$$

$$I_b(t) + \theta I_b(t) = -A e^{b\mu_2}, \mu_2 \leq t \leq \gamma_2 \tag{6}$$

$$I_b(t) = -\frac{A}{1 + \delta(T - t)} e^{b\mu_2}, T \leq t \leq \frac{T_2}{n} \tag{7}$$

With the initial conditions $I_{v1}(0) = 0, I_{v2}(T_2) = 0$ and $I_b(T) = 0$, the solution of the above differential equations are

$$I_{v1}(t_1) = \frac{A(K - 1)}{(b + \theta)} e^b [e^{bt_1} - e^{-\theta t_1}], 0 \leq t_1 \leq \mu_1 \tag{8}$$

$$= \frac{A(K - 1)}{\theta} e^{b\mu_1} + [I_{v1}(\mu_1) - \frac{A(K - 1)}{\theta} e^{b\mu_1}] e^{-\theta(\mu_1 - t_1)}, \mu_1 \leq t_1 \leq \gamma_1 \tag{9}$$

$$I_{v_2}(t_2) = -\frac{A}{\theta + b} e^{bt_2} + \left[I_{v_2}(\mu_2) + \frac{A}{\theta + b} e^{b\mu_2} \right] e^{\theta(\mu_2 - t_2)}, 0 \leq t_2 < \mu_2 \tag{10}$$

$$= \frac{A}{\theta} e^{b\mu_2} [e^{\theta(T_2 - \mu_2)} - 1], \mu_2 \leq t_2 \leq \gamma_2 \tag{11}$$

$$I_b(t) = -\frac{A}{\theta + b} e^{bt} + \left[I_b(\mu_2) + \frac{A}{\theta + b} e^{b\mu_2} \right] e^{\theta(\mu_2 - t)}, 0 \leq t \leq \mu_2 \tag{12}$$

$$= \frac{A}{\theta} e^{b\mu_2} [e^{\theta(T - \mu_2)} - 1], \mu_2 \leq t \leq \gamma_2 \tag{13}$$

$$I_b(t) = \frac{-A}{\delta} e^{-bt} \left\{ \ln \left[1 + \delta \left(\frac{T_2}{n} - T \right) \right] - \ln \left[1 + \delta \left(\frac{T_2}{n} - t \right) \right] \right\}, T \leq t \leq \frac{T_2}{n} \tag{14}$$

From (10), we have

$$I_{mv} = -\frac{A}{\theta + b} + \left[I_{v_2}(\mu_2) + \frac{A}{\theta + b} e^{b\mu_2} \right] e^{\theta\mu_2} \tag{15}$$

Similarly from (12)

$$I_{mb} = -\frac{A}{\theta + b} + \left[I_b(\mu_2) + \frac{A}{\theta + b} e^{b\mu_2} \right] e^{\theta\mu_2} \tag{16}$$

By the boundary condition, $I_{v_1}(T_1) = I_{v_2}(0)$, one can get the relation between T_1 and T_2 .

The yearly holding cost for buyer and vendor is

$$\begin{aligned} HC_b &= p_b F_b \int_0^T I_b(t) dt \\ &= p_b F_b \left[\int_0^{\mu_2} I_b(t) dt + \int_{\mu_2}^T I_b(t) dt \right] \end{aligned} \tag{17}$$

and

$$\begin{aligned} HC_v &= p_v F_v \left[\int_0^{T_1} I_{v_1}(t) dt + \int_0^{T_2} I_{v_2}(t_2) dt_2 - \int_0^T I_b(t) dt \right] \\ &= p_v F_v \left[\int_0^{\mu_1} I_{v_1}(t_1) dt_1 + \int_{\mu_1}^{T_1} I_{v_1}(t_1) dt_1 + \int_0^{\mu_2} I_{v_2}(t_2) dt_2 + \int_{\mu_2}^{T_2} I_{v_2}(t_2) dt_2 - \int_0^{\mu_2} I_b(t) dt - \int_{\mu_2}^T I_b(t) dt \right] \end{aligned} \tag{18}$$

The annual deteriorated costs for buyer and vendor is

$$\begin{aligned} DC_b &= p_b \left(I_{mb} - \int_0^T D(t) dt \right) \\ &= p_b \left[I_{mb} - \left(\int_0^{\mu_2} D(t) dt + \int_{\mu_2}^T D(t) dt \right) \right] \end{aligned} \tag{19}$$

and

$$DC_v = p_v (PT_1 - I_{mb}) \tag{20}$$

respectively

The setup cost per year for buyer and vendor is

$$S C_b = C_{s_b} \dots(21)$$

and

$$S C_v = C_{s_v} \dots(22)$$

respectively

The shortage cost for buyer

$$O C_b = S \int_T^{T_2} -I_b(t) dt \dots\dots(23)$$

The opportunity cost for buyer

$$L C_b = \pi \int_T^{T_2} A \left[1 - B\left(\frac{T_2}{n} - t\right) \right] dt \dots(24)$$

Therefore, the buyer’s cost is the sum of (17), (19), (21), (23) and (24) as

$$B C = H C_b + D C_b + S C_b + O C_b + L C_b \dots(25)$$

The vendor’s cost is the sum of (40), (42) & (44) as

$$V C = H C_v + D C_v + S C_v \dots(26)$$

The integrated total cost of the vendor and buyer, is the sum of (47) and (48)

$$T C = B C + V C \dots(27)$$

4. CONCLUSION

In the present study we have developed a varying demand deteriorating inventory model with a very realistic and practical demand rate. Today, when market trend has changed to such a large extent, it becomes imperative that we take more than one trend in account when considering customer’s demand. The procedure presented here may be applied to very practical situations. To make a better combination of increasing-steady-decreasing demand patterns for perishable seasonal products and finite length of the season this model can be used. The customer neither has the patience nor the requirement to wait. This often results in lost sales. As we compare both models we have seen that total cost without shortages is very high in comparison of with shortages.

An optimal solution of the system is obtained under the assumed conditions. Moreover, we characterize the effects of various parameters of the system on the optimal solution.

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